Math 60 7.2 Multiplying and Dividing Rational Expressions

Objectives:

1) Multiply two rational expressions
   a. Factor everything completely
   b. Divide out common factors
2) Divide two rational expressions
   a. Rewrite as multiply by reciprocal of the second fraction
   b. Factor everything completely
   c. Divide out common factors

Practice:

Perform the indicated operations and simplify.

1) $\dfrac{p^2-9}{p^2+5p+6}\cdot\dfrac{p+2}{6-2p}$

2) $\dfrac{m^2-n^2}{10m^2-10mn}\cdot\dfrac{10m+5n}{2m^2+3mn+n^2}$

3) $\dfrac{4-z^2}{3z-2}\cdot\dfrac{3z^2+7z-6}{z^2+z-6}$

4) $\dfrac{n^2-n-2}{4n^2-9}\cdot\dfrac{2n^2-n-6}{2n^2-5n+3}$

5) $\dfrac{8n-8}{n^2-3n+2}\cdot\dfrac{2n^2+9n+10}{12}$

6) $\dfrac{y^2-9}{2y^2-y-15}\div\dfrac{3y^2+10y+3}{2y^2+y-10}$

7) $\dfrac{a^2+3a+2}{a^2-9}\div(a+2)$

8) $\dfrac{\dfrac{12x}{5x+20}}{\dfrac{4x^2}{x^2-16}}$

9) $\dfrac{\dfrac{x^2+25}{x^7}}{\dfrac{x^2+5x}{x^2}}$

10) $\dfrac{x^2-5x}{x^3+125}\div\dfrac{x-5}{x^2-5x+25}$

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① $\frac{p^2-9}{p^2+5p+6} \cdot \frac{p+2}{6-2p}$

$= \frac{(p+3)(p-3)}{(p+2)(p+3)} \cdot \frac{(p+2)}{(-2)(p-3)}$

$= \frac{1}{-2}$

$= \boxed{\frac{-1}{2}}$

Factor completely

1) $p^2-9 = (p-3)(p+3)$

2) p^2+5p+6 (6: 2, 3; 5) $= (p+2)(p+3)$

3) $p+2$ already factored $(p+2)$

4) $6-2p = -2p+6$ standard form
$= -2(p-3)$ GCF w/ neg.

Cancel common factors

$\frac{p+3}{p+3} \cdot \frac{p-3}{p-3} \cdot \frac{p+2}{p+2}$

② $\frac{m^2-n^2}{10m^2-10mn} \cdot \frac{10m+5n}{2m^2+3mn+n^2}$

$= \frac{(m+n)(m-n)}{10m(m-n)} \cdot \frac{5(2m+n)}{(m+n)(2m+n)}$

$= \frac{5}{10m}$

$= \boxed{\frac{1}{2m}}$

Factor completely.

1) $m^2-n^2 = (m-n)(m+n)$
difference of two squares

2) $10m^2-10mn = 10m(m-n)$
GCF $10m$

3) $10m+5n = 5(2m+n)$
GCF $= 5$

4) $2m^2+3mn+n^2$ ($2m^2n^2$; $1mn$, $2mn$; $3mn$)

$= 2m^2+2mn+1mn+n^2$

$= 2m(m+n)+n(m+n)$

$= (m+n)(2m+n)$

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③ $\frac{4-z^2}{3z-2} \cdot \frac{3z^2+7z-6}{z^2+z-6}$

$= \frac{-(z-2)(z+2)}{(3z-2)} \cdot \frac{(z+3)(3z-2)}{(z+3)(z-2)}$

$= -(z+2)$

$= \boxed{-z-2}$

Final answer is not a fraction - Simplify by distribute.

If final answer is a fraction, leave factored

Factor completely.

1) $4-z^2$

$= -z^2+4$ standard form

$= -(z^2-4)$ GCF = -1

$= -(z-2)(z+2)$ diff of two squares

⇑ keep the (-1) in the answer.

2) $3z-2$ already factored

$=(3z-2)$

3) $3z^2+7z-6$ (-18; 9, -2; 7)

$= 3z^2+9z-2z-6$

$= 3z(z+3)-2(z+3)$

$= (z+3)(3z-2)$

4) z^2+z-6 (-6; 3, -2; 1)

$(z+3)(z-2)$

④ $\frac{n^2-n-2}{4n^2-9} \cdot \frac{2n^2-n-6}{2n^2-5n+3}$

$= \frac{(n-2)(n+1)}{(2n-3)(2n+3)} \cdot \frac{(n-2)(2n+3)}{(2n-3)(n-1)}$

$= \frac{(n-2)(n+1)(n-2)}{(2n-3)(2n-3)(n-1)}$

$= \boxed{\frac{(n+1)(n-2)^2}{(n-1)(2n-3)^2}}$

Leave answer factored

Note: If the similar factors are both in the numerator (or both in the denominator) they cannot be canceled.

1) n^2-n-2 (-2; -2, +1; -1)

$(n-2)(n+1)$

2) $4n^2-9$ diff of sq.

$=(2n-3)(2n+3)$

3) $2n^2-n-6$ (-12; -4, +3; -1)

$= 2n^2-4n+3n-6$

$= 2n(n-2)+3(n-2)$

$= (n-2)(2n+3)$

4) $2n^2-5n+3$ (6; -3, -2; -5)

$= 2n^2-3n-2n+3$

$= n(2n-3)-1(2n-3)$

$= (2n-3)(n-1)$

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(5) $\frac{8n-8}{n^2-3n+2} \cdot \frac{2n^2+9n+10}{12}$

$= \frac{8(n-1)}{(n-2)(n-1)} \cdot \frac{(n+2)(2n+5)}{12}$

$= \frac{8}{12} \cdot \frac{(n+2)(2n+5)}{(n-2)}$

$= \boxed{\frac{2(n+2)(2n+5)}{3(n-2)}}$

Note: $(n+2)$ and $(n-2)$ are two different numbers.
For example: If $n=3$
$n+2 \Rightarrow 3+2=5$
$n-2 \Rightarrow 3-2=1$

1) $8n-8 = 8(n-1)$ GCF = 8

2) n^2-3n+2 (2, −1; −2, −1; −3)
$(n-2)(n-1)$

3) $2n^2+9n+10$ (20; 4, 5; 9)

$= 2n^2+4n+5n+10$
$= 2n(n+2)+5(n+2)$
$= (n+2)(2n+5)$

4) 12

multiply by reciprocal

(6) $\frac{y^2-9}{2y^2-y-15} \div \frac{3y^2+10y+3}{2y^2+y-10}$

$= \frac{y^2-9}{2y^2-y-15} \cdot \frac{2y^2+y-10}{3y^2+10y+3}$

$= \frac{(y-3)(y+3)}{(y-3)(2y+5)} \cdot \frac{(2y+5)(y-2)}{(y+3)(3y+1)}$

$= \boxed{\frac{y-2}{3y-1}}$

factor completely

1) $y^2-9 = (y-3)(y+3)$
difference of squares

2) $2y^2-y-15$ (−30; −6, +5; −1)

$= 2y^2-6y+5y-15$
$= 2y(y-3)+5(y-3)$
$= (y-3)(2y+5)$

3) $2y^2+y-10$ (−20; 5, −4; 1)
$= 2y^2+5y-4y-10$
$= y(2y+5)-2(2y+5)$
$= (2y+5)(y-2)$

4) $3y^2+10y+3$ (9; 9, 1; 10)
$= 3y^2+9y+y+3$
$= 3y(y+3)+1(y+3)$
$= (y+3)(3y+1)$

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(7) $\dfrac{a^2+3a+2}{a^2-9} \div (a+2)$

$= \dfrac{a^2+3a+2}{a^2-9} \cdot \dfrac{1}{(a+2)}$ — multiply by reciprocal

$= \dfrac{\cancel{(a+2)}(a+1)}{(a+3)(a-3)} \cdot \dfrac{1}{\cancel{(a+2)}}$ — factor completely

1) $a^2+3a+2 = (a+2)(a+1)$
2) $a^2-9 = (a-3)(a+3)$
3) 1
4) $(a+2)$

$= \boxed{\dfrac{(a+1)}{(a+3)(a-3)}}$

Recall: Leave fractions factored.

(8) $\dfrac{\dfrac{12x}{5x+20}}{\dfrac{4x^2}{x^2-16}}$ ← This division bar becomes ÷ symbol.

$= \dfrac{\left(\dfrac{12x}{5x+20}\right)}{\left(\dfrac{4x^2}{x^2-16}\right)}$

$= \dfrac{12x}{5x+20} \div \dfrac{4x^2}{x^2-16}$ — write with ÷ symbol

$= \dfrac{12x}{5x+20} \cdot \dfrac{x^2-16}{4x^2}$ — multiply by reciprocal

$= \dfrac{12 \cdot x}{5\cancel{(x+4)}} \cdot \dfrac{(x-4)\cancel{(x+4)}}{4x^2}$ — factor completely

$= \dfrac{12 \cdot x \cdot (x-4)}{4 \cdot 5 \cdot x^2}$ — reduce $\frac{12}{4} = 3$, $\frac{x}{x^2} = \frac{1}{x}$

$= \boxed{\dfrac{3(x-4)}{5x}}$ — leave result factored

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(9) $\dfrac{\dfrac{x^2+25}{x^7}}{\dfrac{x^2+5x}{x^2}}$

$= \dfrac{x^2+25}{x^7} \div \dfrac{x^2+5x}{x^2}$ rewrite as division

$= \dfrac{(x^2+25)}{x^7} \cdot \dfrac{x^2}{x^2+5x}$ rewrite as multiply by reciprocal of 2nd fraction

$= \dfrac{(x^2+25)}{x^7} \cdot \dfrac{x^2}{x(x+5)}$ sum of squares x^2+25 is prime; factor GCF x from x^2+5x

$= \dfrac{x^2}{x^8} \cdot \dfrac{(x^2+25)}{(x+5)}$ rewrite like bases near each other.

$= \boxed{\dfrac{(x^2+25)}{x^6(x+5)}}$ exponent law

(10) $\dfrac{x^2-5x}{x^3+125} \div \dfrac{x-5}{x^2-5x+25}$

$= \dfrac{x^2-5x}{x^3+125} \cdot \dfrac{x^2-5x+25}{x-5}$ rewrite as multiply by reciprocal of 2nd fraction

$= \dfrac{x(x-5)}{(x+5)(x^2-5x+25)} \cdot \dfrac{(x^2-5x+25)}{(x-5)}$ factor GCF x from x^2-5x; factor sum of cubes x^3+125; add parentheses to prime polynomials $(x^2-5x+25)$ and $(x-5)$

$= \dfrac{x\,(x-5)(x^2-5x+25)}{(x-5)(x^2-5x+25)(x+5)}$ write like bases near each other & cancel common factors

$= \boxed{\dfrac{x}{x+5}}$